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18 June 2002

SUBJECT: Authorization for Release of Technical Information, Control Number: **AFRL-PR-ED-TP-2002-154**  
D.E. Kirtley (ERC) et al., "Analysis of Xenon Flow Calibration Techniques for Electric Thruster Testing"

AIAA JPC

(Statement A)

(Indianapolis, IN, 8-10 July 2002) (Deadline = 08 Jul 02)

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Technical Advisor  
Space and Missile Propulsion Division

# Analysis of Xenon Flow Calibration Techniques for Electric Thruster Testing

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In assessing the performance characteristics of Hall-effect thrusters (HETs) and ion engines, it is important to have an accurate estimate of the propellant flow rate to the engine. The difficulty lies in accurately measuring the very low flow rates, below 1 mg/s of xenon gas. Most test facilities use commercial calibration-certified mass flow controllers and meters. However, these flow meters drift over time from their calibration. Recalibration is typically an expensive and time consuming task which can only be performed at a few select locations; therefore it is desirable for an in-house calibration verification system to be available to detect calibration drift. Here we examine the theory of constant-volume calibration, and propose a cost effective constant-volume flow rate verification apparatus.

## Introduction

The Air Force Research Laboratory (AFRL) at Edwards AFB has completed construction of a vacuum test facility designed for long-duration testing of Hall-effect thrusters (HETs) up to 5 kW in power. Facility requirements dictate that all independent performance measurements be made to within 1% of nominal conditions. Test measurements made during thruster operation include thrust, power consumption, chamber pressure and propellant usage (mass flow rate). To provide propellant to the chamber a Propellant Feed System (PFS) was constructed and mounted to the side of the chamber. The PFS provides clean and accurately metered high purity xenon gas to the thruster through the chamber wall.

In designing the PFS, AFRL engineers discovered that a number of HET test facilities around the country used different techniques to calibrate their propellant flow measurement systems. Most techniques appeared to provide an acceptable result. However, it became clear that these differences needed to be discussed.

There are several options for calibration of flow measurement systems. Of the options available, the most commonly used methods employ a piston prover, direct mass measurement, or constant volume. In the piston prover method, flow rate is determined by measurement of the rate of change of volume reservoir, typically at constant pressure. Direct mass measurement involves simply weighing a volume of gas before and after the experiment. In constant volume methods, flow rate is determined by measuring the rate of change of pressure in a constant volume reservoir.

Constant volume calibration requires minimal equipment, and can be configured as a fixed, in-situ apparatus built into the PFS. However, it does require some knowledge of gas behavior (a gas law, or equation of state) for the range of pressures measured. This paper discusses the importance of the gas law in constant-volume calibration, and proposes some procedures for accurately verifying xenon flow meter calibration.

## Theory

The Ideal Gas Law is often used in in-situ calibration schemes. It can be derived by assuming that the molecules that make up the gas have negligible sizes, that their collisions with themselves and the wall are perfectly elastic, and that the molecules have no interactions with each other.

$$PV = nRT \quad (1)$$

Above,  $P$  is the pressure,  $V$  is the volume,  $n$  is the amount of the gas (in moles),  $R$  is the gas constant (8314.5 J/mol/K), and  $T$  is the temperature. (All units in this paper can be assumed to be SI, unless otherwise specified.) The Ideal Gas Law does not account for intermolecular forces.

Van der Waals proposed that we correct for the compressibility of the gas by subtracting a term,  $b$ , from the volume of the real gas before we substitute it into the ideal gas equation to account for the fact that the volume of a real gas is too large at high pressures. To correct for the fact that the pressure of a real gas is smaller than expected from the ideal gas equation, van der Waals added a second term,  $a$ , to

the pressure in this equation. The complete van der Waals equation is therefore:

$$\left(P + \frac{n^2 a}{V_m^2}\right)(V_m - nb) = nRT \quad (2)$$

For xenon,  $a$  and  $b$  have been determined to be 4.192 bar L<sup>2</sup>/mol and 0.05156 L/mol respectively.<sup>1</sup>

Two other real gas approximations are the Berthelot and Redlich-Kwong equations. Berthelot was modified for the temperature dependence of the attractive term and has higher accuracy at low temperatures. The Redlich & Kwong equation incorporates the temperature and excluded volume into the attractive term and is considered the most complicated and accurate two-parameter equation of state for real gas. In the pressure and temperature range of interest here, Berthelot and Redlich-Kwong agree strongly with van der Waals. Therefore, in the following analysis, we will treat van der Waals as "exact."

In both the Ideal Gas Law and van der Waals, the terms may be rearranged such that  $n$  and  $V$  appear as  $n/V$ , or gas density. Mass density,  $\rho$ , is  $\rho = nM/V$ , where  $M$  is the molecular weight. Plots of  $\rho$  versus  $P$  are shown in Fig. 1. To obtain this plot for van der Waals, a numerical root-finder was used.

In order to measure the instantaneous flow rate of gas into a constant-volume cylinder at constant temperature, the rate of pressure change of the cylinder must be measured. Then, the following relationship may be applied:

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial P} \frac{dP}{dt} = f(P) \frac{dP}{dt} \quad (3)$$

which, for constant volume is the mass flow rate into the volume, divided by the volume. Above,  $f(P)$  is the slope of the density-pressure curve in Fig. 1. The Ideal Gas Law gives simply,

$$f_{ideal}(P) = \frac{M}{RT} \quad (4)$$

The expression for  $f(P)$  is not as straightforward for the van der Waals equation. However, it may be found by numerical differentiation. The results are shown in Fig. 2.

To accurately estimate  $f(P)$  for a constant-volume calibration, one could use  $f_{ideal}(P) + C$ , where  $C$  is a correction factor. This would be accurate for determining flow rate at one pressure value, but may not be suitable for determination of instantaneous flow rate over a range of pressures.

It appears that  $f_{vdw}(P)$  is nearly linear in  $P$  in the range of interest. Therefore, a good linear approximation to  $f_{vdw}(P)$  may be used that is more accurate:

$$f_{vdw}(P) \cong (1 + 1.12 \times 10^{-7} P) \frac{M}{RT} \quad (5)$$

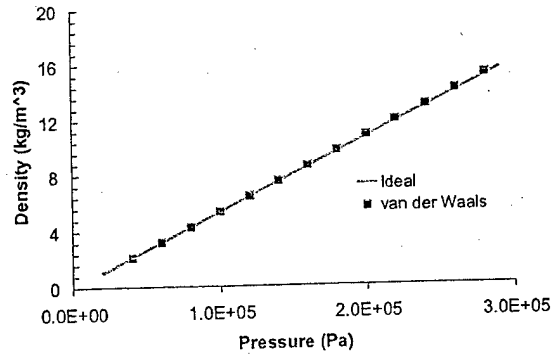


Fig. 1. Comparison of density versus pressure for Ideal and van der Waals equations of state.  $T = 295.4$  degK

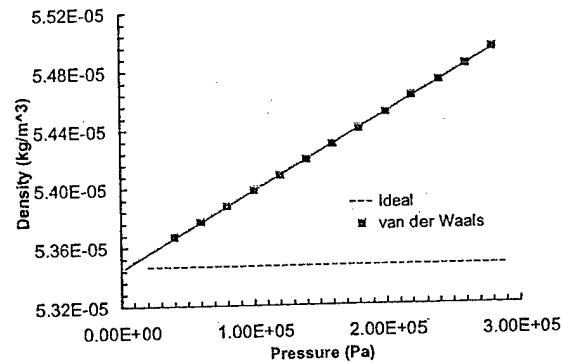


Fig. 2. Comparison of  $f(P)$  for Ideal and van der Waals equations of state.  $T = 295.4$  degK.

### Suggested Flow Rate Calibration Procedure

To estimate flow rate of xenon into a constant volume, one may accurately record the time history of pressure and temperature in the volume. The temperature measurement may be used to verify that temperature of the volume is, indeed, constant during the test. Then, one may apply Eqns. (3) and (5) to determine time rate of change of density in the volume. Finally, multiplying by the volume yields the mass flow rate. In summary:

$$\dot{m} = V \frac{d\rho}{dt} = \frac{MV}{RT} (1 + 1.12 \times 10^{-7} P) \frac{dP}{dt} \quad (6)$$

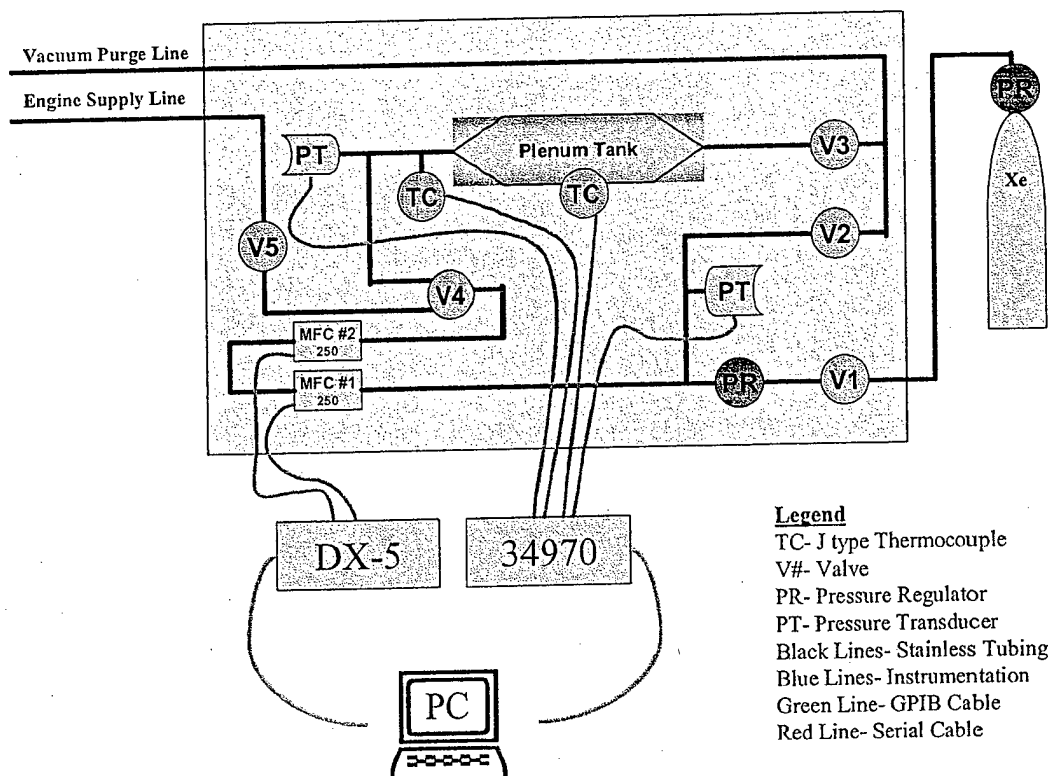


Fig. 3. Diagram of AFRL Chamber 3 in-situ flow calibration system.

For xenon, this is,

$$\dot{m} = \frac{V}{(63.325)T} (1 + 1.12 \times 10^{-7} P) \frac{dP}{dt} \quad (7)$$

Again, all units in Eq. (7) are SI.

### Measurement Uncertainties

The focus of this investigation is the error in the in-situ mass flow measurement. Therefore, one must look at potential error in the calculations as well as in the measurement devices. The errors inherent in the measurement devices, and transient noise in the acquisition of these measurements, may contribute to uncertainty in the final estimate of flow rate. Since the behavior of the Ideal Gas Law and van der Waals is, in general, similar, we will base the uncertainty estimate on the Ideal Gas Law. Using techniques commonly used, an analytical solution can be found:<sup>2</sup>

$$\frac{\delta \dot{m}}{|\dot{m}|} = \sqrt{\left[ \left( \frac{\delta V}{|V|} \right)^2 + \left( \frac{\delta T}{|T|} \right)^2 \right] + \frac{\delta P_2^2 + \delta P_1^2}{|P_2 - P_1|^2} + \frac{\delta t_2^2 + \delta t_1^2}{|t_2 - t_1|^2}} \quad (8)$$

In the above notation,  $\delta x$  represents uncertainty in value  $x$ .

### Theoretical Uncertainties

As can be seen in Fig. 2, the state derivative,  $f(P)$ , differs between the Ideal Gas Law and van der Waals by up to 5% for typical pressures in HET feed systems. Therefore, when applying the calibration procedure suggested above using  $f(P) = f_{\text{Ideal}}(P)$ , errors in mass flow rate of up to 5% can be expected, depending on the pressure. Use of Eq. (7), as suggested, can reduce this error to less than 0.1%.

### Apparatus

An experiment was performed at AFRL to apply the calibration technique described above. A diagram of the apparatus is shown in Fig. 3.

The apparatus was designed to run an experimental HET and is capable of controlling/metering two separate cathodes (20 sccm max.) and one anode (250 sccm max.). The PFS consists of polished stainless steel tubing orbitally welded with VCR fittings and NuPro metal seals. Polished stainless steel gas flow valves are used to control the propellant flow path. Two MKS Baratron® capacitance monometers monitor pressure at separate locations within the pipeline. Two Unit Instruments 8160 MFCs (in series) monitor and/or control the propellant

flow while a DX-5 controller acts as the computer interface between the MFCs and the data acquisition computer. Data acquisition and control is maintained by a Dell Dimension XPS T500 desktop computer operating Windows 98® and LabView 5.1®. An Agilent 34970A datalogger with a 34901A 20 channel multiplexer card and a 34903A 20 channel switch actuator controls the switches the valves and measures the pressure. The data acquisition computer uses a serial interface for communication with the DX-5 and GPIB for communication with the Agilent data logger.

Flow measurements are made using the Unit Instruments 8160 Mass Flow Controller (MFC). These instruments may be used for both flow control and flow metering. They have been calibrated by the manufacturer using high purity Xenon gas (99.994%). The Model 8160 has an ultra clean all stainless steel design flow path.

### Test Procedure

Test procedure was as follows:

- 1.) Close all valves and set the flow controllers to 0% flow. This stops any flow within the panel and ensures that the auto zero feature on the MFCs is initiated.
- 2.) Open valve V3 to dump the plenum tank into the vacuum chamber. Once the pressure has fallen to 0 Pa close the valve.
- 3.) Set the flow controller you wish to verify to 125% of normal flow. Set the other to the desired flow rate.
- 4.) Open valves V1 and V5 to begin steady state flow into the vacuum chamber (this usually only takes 1 minute). V4 is a 3-way valve and is default open to allow flow to V5. V5 allows gas flow into the chamber and to the HET.
- 5.) Once steady state flow is achieved V4 is switched to allow flow to begin entering the plenum tank.
- 6.) After the plenum tank reaches the required pressure all valves are closed and data recording stops.

### Experimental Results

Several runs were performed at various flow rates. Fig. 4 shows the results with the flow controller set to 24.0 mg/s. It shows the resulting time history of pressure rise in the calibration volume, and the corresponding calculated xenon flow rate (using Eq. (7)(8)). In this case, the calculated flow rate agreed with the flow controller reading to within 2%.

For comparison purposes, flow rate was determined using three different equations of state: the Ideal Gas Law, van der Waals, and Berthelot. The results are shown in Fig. 5. As suggested by the theoretical analysis in the preceding sections, using the Ideal Gas Law results in under prediction of flow rate by 4% at 50 psi.

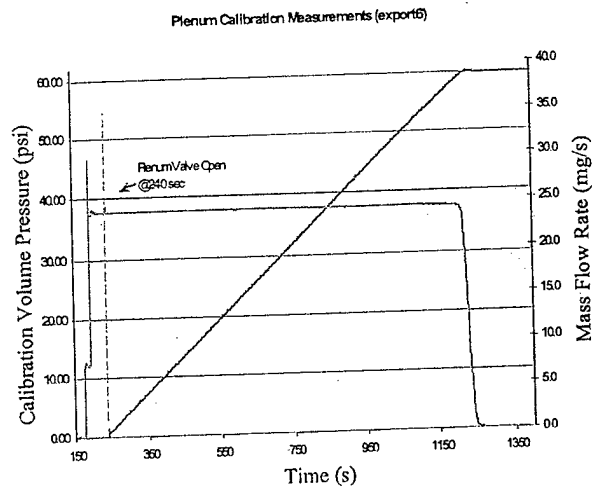


Fig. 4. Time history of pressure rise in constant volume, and corresponding xenon flow rate calculated.

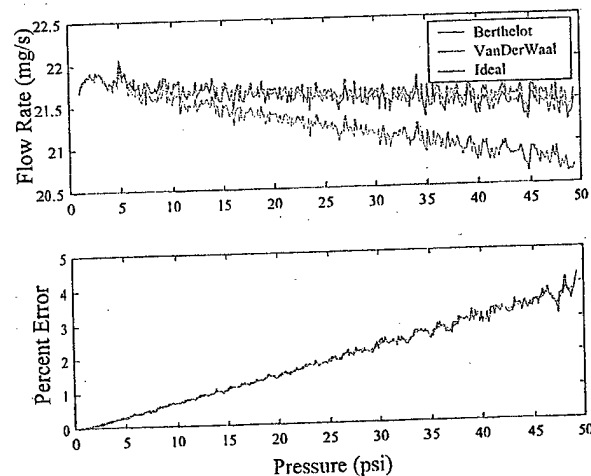


Fig. 5. Flow rate calculated versus pressure using various equations of state. Lower figure shows the percent error between flow rate calculated using Ideal Gas Law versus van der Waals.

### Conclusions

In-situ calibration of low-rate xenon flow systems can be done relatively easily and cheaply. The method presented here involves measuring the rate of change of pressure in a calibration volume, then using an equation of state to determine the mass flow rate.

The choice of which equation of state to use may significantly affect the results. At 50 psi, the mass flow rate calculations differ by 4% based on which equation of state is used – Ideal

Gas Law or van der Waals. However, the van der Waals equation is highly nonlinear and difficult to use. Therefore, a simple linear fit to van der Waals was derived for xenon in the pressure range 0-50 psi. This fit can be used to increase the accuracy of in-situ constant volume calibrations without requiring complicated numerical root finding. The result is a straightforward in-situ calibration system that is accurate and easy to use.

### References

<sup>1</sup>Reid, R.C., Prausnitz, J.M., and Poling, B.E., *The Properties of Gases and Liquids*, Fourth Edition, McGraw-Hill, New York, 1987.

<sup>2</sup>American National Standard for Expressing Uncertainty - *U.S. Guide to the Expression of Uncertainty in Measurement*, ANSI/NCSL Z540-2-1997, Oct. 1997.